Pion Mass Difference in Bars-Halpern-Yoshimura Unified Gauge Model

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Pion mass difference has been calculated in the renormalizable gauge model of Bars-Halpern-Yoshimura. It is found that the soft pion mass difference is finite in the lowest order perturbative calculation, similar to the result obtained in the unified model of Salam-Weinberg. The mass difference of Kaon has also been discussed.

I. Introduction

Recent developments of gauge theories for the unification of the weak, electromagnetic, and strong interactions have led to several calculations on the pion mass difference [1–4]. The electromagnetic mass difference of hadrons has also been discussed in the literature [5]. Working in the Salam-Weinberg unified model [6], it has been shown by Dicus and Mathur [1] that the pion mass difference comes out to be finite in the soft pion limit and the divergences associated with photon and Z-meson exchange cancel without using the Weinberg second spectral sum rule. We investigate the same problem in the unified gauge model proposed by Bars-Halpern-Yoshimura (BHY)[7]. However, instead of using current algebra techniques as used earlier [1–5], we compute the soft pion mass difference in the lowest order perturbative approach. We shall show that the pion mass difference turns out to be finite in the soft pion limit. In Sect. II we shall construct interaction Lagrangian, in Sect. III we shall calculate mass difference and in Sect. IV we shall discuss the results.

II. SU(2) ⊗ SU(2) Lagrangian

It is a characteristic feature of the BHY model that a natural $(3,\overline{3}) \oplus (\overline{3},3)$ symmetry breaking emerges for the hadrons, and that the neutral $\Delta S = 1$ currents are suppressed without enlarging the number of quarks. The appropriate SU(2) ⊗SU(2) form of the BHY Lagrangian density is

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}\,\mathrm{Tr}\,[(\nabla_\mu M_\mathrm{L})^+\nabla_\mu M_\mathrm{L} + (\nabla_\mu M_\mathrm{R})^+\nabla_\mu M_\mathrm{R}] + \frac{\lambda_1}{4!}\,[(\mathrm{Tr}\,M_\mathrm{L} M_\mathrm{L}^+)^2 + (\mathrm{Tr}\,M_\mathrm{R} M_\mathrm{R}^+)^2] \\ &+ \frac{\lambda_2}{4!}\,[\mathrm{Tr}\,(M_\mathrm{L} M_\mathrm{L}^+)^2 + \mathrm{Tr}\,(M_\mathrm{R} M_\mathrm{R}^+)^2] - \frac{1}{2}\,\mathrm{Tr}\,[(\nabla_\mu \Sigma)^+\nabla_\mu \Sigma] + \frac{\lambda_3}{4!}\,[\mathrm{Tr}\,(\Sigma\Sigma^+)^2] \\ &+ \mathcal{L}_1(L_\mu, R_\mu) + \mathcal{L}_2(N, L_\mu, R_\mu) + \mathcal{L}_3(N, \Sigma, M_\mathrm{L}, M_\mathrm{R})\,.\end{aligned} \tag{1}$$

The covariant derivatives for nucleons, Σ and M hadrons are defined by

$$\begin{aligned}&\nabla_\mu \begin{Bmatrix} N_\mathrm{L} \\ N_\mathrm{R}\end{Bmatrix} \equiv \partial_\mu \begin{Bmatrix} N_\mathrm{L} \\ N_\mathrm{R}\end{Bmatrix} - i f \begin{Bmatrix} L_\mu \\ R_\mu\end{Bmatrix}\left[T, \begin{Bmatrix} N_\mathrm{L} \\ N_\mathrm{R}\end{Bmatrix}\right] \quad N_\mathrm{L} \equiv \frac{1+\gamma_5}{2}\begin{pmatrix}\mathrm{p}\\ \mathrm{n}\end{pmatrix}, \quad N_\mathrm{R} \equiv \frac{1-\gamma_5}{2}\begin{pmatrix}\mathrm{p}\\ \mathrm{n}\end{pmatrix} \\ &\nabla_\mu \begin{Bmatrix} \Sigma \\ M_\mathrm{L} \\ M_\mathrm{R}\end{Bmatrix} \equiv \partial_\mu \begin{Bmatrix} \Sigma \\ M_\mathrm{L} \\ M_\mathrm{R}\end{Bmatrix} + i f \begin{Bmatrix} L_\mu \Sigma \\ L_\mu M_\mathrm{L} \\ R_\mu M_\mathrm{R}\end{Bmatrix} - i \begin{Bmatrix} f\,\Sigma R_\mu \\ g\,M_\mathrm{L} W_\mu \\ g_{1/2} B_\mu M_\mathrm{R} \tau_3\end{Bmatrix}\end{aligned} \tag{2}$$

where $\Sigma \equiv (\sigma \hat{l} + i\boldsymbol{\pi}, \boldsymbol{T})$ and $(R_\mu \pm L_\mu)$ stand for vector and axialvector mesons. Introducing the vacuum expectation values and $\langle M_{\mathrm{L,R}}\rangle = \langle M^+_{\mathrm{L,R}}\rangle \equiv \langle\varkappa\rangle$, which are required for the spontaneous symmetry breakdown and the Higgs-Kibble phenomenon [8] in (1), we generate direct coupling between the strong and weak gauge bosons via vector-meson dominance. The relevant interaction Lagrangian density is thus obtained [9]

$$\begin{aligned}\mathcal{L}_\mathrm{int} = &+\tfrac{1}{2} f g \langle\varkappa\rangle^2 [\varrho_\mu^+ W_\mu^- + \varrho_\mu^- W_\mu^+ - A_\mu^+ W_\mu^- - A_\mu^- W_\mu^+] \\ &+ \frac{f\langle\varkappa\rangle^2}{2(g^2+g'^2)^{1/2}}\,[(g^2-g'^2)\,\varrho_\mu^0 Z_\mu - (g^2+g'^2)\,A_\mu^0 Z_\mu] \\ &+ e f \langle\varkappa\rangle^2 \varrho_\mu^0 A_\mu^{(\gamma)} + i f\,[\pi^- \overleftrightarrow{\partial}_\mu \pi^0 \varrho_\mu^+ + \pi^+ \overleftrightarrow{\partial}_\mu \pi^- \varrho_\mu^0 + \pi_0 \overleftrightarrow{\partial}_\mu \pi^+ \varrho_\mu^- + \pi^0 \overleftrightarrow{\partial}_\mu \sigma A_\mu^0] \\ &+ i f^2 \langle\sigma\rangle\,[+\pi^0 \varrho_\mu^- A_\mu^+ - \pi^0 \varrho_\mu^+ A_\mu^- - \pi^+ \varrho_\mu^- A_\mu^0 - \pi^- \varrho_\mu^0 A_\mu^+ - \pi^+ \varrho_\mu^0 A_\mu^- - \pi^- \varrho_\mu^+ A_\mu^0]\end{aligned} \tag{3}$$

in which the coupling constants and the parameters are expressed in terms of known quantities

$$f^2\langle\varkappa\rangle^2 = m_\varrho^2\,, \quad g\,g'/(g^2+g')^{1/2} = e = g\sin\Theta_\mathrm{W} = g'\cos\Theta_\mathrm{W}$$

$$f^2[\langle\varkappa\rangle^2 + 2\langle\sigma\rangle^2] = m_{A_1}^2\,, \quad g^2/8\,m_\mathrm{W}^2 = G_\mathrm{F}/\sqrt{2}\,, \quad m_\mathrm{Z} = m_\mathrm{W}/\cos\Theta_\mathrm{W}\,.$$

III. Pion Mass Difference

To obtain the mass difference between charged and neutral pions in a perturbative way, we compute the matrix elements of various lowest order self energy diagrams of π^+ and π^0 due to the emission and absorption of neutral intermediate vector bosons Z, in addition to the usual photon contribution to the π^+ meson. The diagrams arising from charged vector boson ($W^\pm$) exchanges are not contributing at all to the pion mass difference as they cancel exactly among themselves, similar to Reference 1. It is clear from Fig. 1 that in the coupling between the strong and weak gauge boson a mixing propogator $\varrho\gamma$ is appearing along with the usual vector-meson dominance diagrams.

The finite and divergent parts of the matrix elements are separated by using the dimensional regularization technique of 't Hooft and Veltman[10]. We get finally the contribution of photon and Z-meson exchange diagrams:

Photon exchange

$$\text{Divergent part:} \quad \Delta m_\pi{}^2 = \frac{i\, e^2}{8\,\pi^2}\, m_\pi{}^2 \left(\frac{2}{4-n}\right), \tag{4}$$

$$\text{Finite part:} \quad \Delta m_\pi{}^2 = \frac{3\,\alpha}{4\,\pi}\, m_\varrho{}^2\, (2 \ln 2)\,. \tag{5}$$

Z-meson exchange

$$\text{Divergent part:} \quad \Delta m_\pi{}^2 = \frac{i\, e^2}{12\,\pi^2}\, \frac{m_\pi{}^4}{m_Z{}^2} \left(\frac{2}{4-n}\right), \tag{6}$$

$$\text{Finite part:} \quad \Delta m_\pi{}^2 \sim O\left(\frac{m_\varrho{}^2}{m_Z{}^2} \ln \frac{m_Z{}^2}{m_\varrho{}^2}\right). \tag{7}$$

The expressions in Eqs. (4) – (7) have been obtained in simplified form as we have used the relation $m_{A_1}^2 \cong 2\, m_\varrho{}^2$. The finite part in $\Delta m_\pi{}^2$ is coming only from Figure 1 (b).

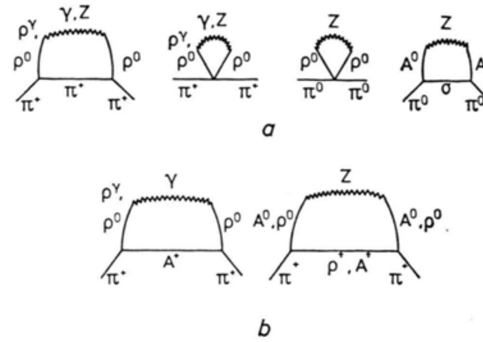

Fig. 1. Lowest order self-energy diagrams for π^+ and π^0. (a) : Chiral symmetry. (b) : Chiral symmetry breaking.

IV. Results

It is clear that the divergent contribution to the pion mass difference [Eq. (4) and (6)] vanishes identically in the soft pion limit ($m_\pi \to 0$). Thus the finite mass difference in the lowest order approximation becomes

$$\Delta m_\pi{}^2 \cong \frac{3\,\alpha}{4\,\pi}\, m_\mathrm{p}{}^2\, (2 \ln 2) + O\left(\frac{m_\varrho{}^2}{m_Z{}^2} \ln \frac{m_Z{}^2}{m_\varrho{}^2}\right)$$
$$\sim 4{\cdot}5 \text{ MeV}$$

which is of the same order as that obtained by previous authors[11, 1–5]. It is to be noted that mass differences of other hadrons can also be calculated from this model. We found from our preliminary calculation that the kaon mass difference comes out in the correct order, but here tadpole diagrams are playing an important role[12].

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